

## TOTAL ENERGIES AND THERMODYNAMIC AND KINETIC DATA

In addition to molecular geometry, energy is certainly the most important quantity to come out of molecular modeling. Energy can be used to reveal which of several isomers is most stable, to determine whether a particular chemical reaction will have a thermodynamic driving force (an *exothermic* reaction) or be thermodynamically uphill (an *endothermic* reaction), and to ascertain how fast a reaction is likely to proceed. Other molecular properties, such as dipole moment, and infrared, Raman, UV/visible and most importantly NMR spectra are also of great interest, but energy plays a special role.

There is more than one way to express the energy of a molecule. Most common to chemists is the heat of formation,  $\Delta H_f$ . This is the heat of a hypothetical chemical reaction that creates a molecule from well defined (but arbitrary) standard states of each of its constituent elements. Note that the heat of formation, which most commonly assumes a value between -1,000 and +500 kJ/mol, cannot be directly measured, but must be obtained indirectly.

An alternative, total energy, is the heat of a hypothetical reaction that creates a molecule from a collection of separated nuclei and electrons. Like the heat of formation, total energy cannot be measured directly, and is used solely to provide a standard method for expressing and comparing energies. Total energies are always negative numbers and are much larger than the sum of the bond energies. By convention, they are expressed in so-called atomic units\* or au, but may be converted to other units as desired:

$$1 \text{ au} = 2625 \text{ kJ/mol}$$

It makes no difference which reference reaction (heat of formation or total energy) is used to calculate the thermochemistry of a balanced chemical reaction (reactant 1 + reactant 2 + ...  $\rightarrow$  product 1 + product 2 + ...):

$$\Delta E(\text{reaction}) = E_{\text{product 1}} + E_{\text{product 2}} + \dots - E_{\text{reactant 1}} - E_{\text{reactant 2}} - \dots$$

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\* The exact energy of hydrogen atom is -0.5 atomic units.

Total energies will be used in the discussion that follows. A negative  $\Delta E$  indicates an *exothermic* (thermodynamically favorable) reaction, while a positive  $\Delta E$  an *endothermic* (thermodynamically unfavorable) reaction.

A special case involves energy differences among isomers. Each comparison may be viewed as a chemical reaction involving only two molecules:

$$\Delta E(\text{isomer}) = E_{\text{isomer 2}} - E_{\text{isomer 1}}$$

A negative  $\Delta E$  means that isomer 2 is more stable than isomer 1.

Total energies may also be used to calculate activation energies,  $\Delta E^\ddagger$ :

$$\Delta E^\ddagger = E_{\text{transition state}} - E_{\text{reactant 1}} - E_{\text{reactant 2}} - \dots$$

Here,  $E_{\text{transition state}}$  is the total energy of the transition state, and  $E_{\text{reactant1}}$ ,  $E_{\text{reactant2}}$ , ... are the total energies of the reactants. Activation energies are expected to be positive numbers\*, meaning that the transition state is less stable than reactants.

Reaction and activation energies are sufficient to know whether a reaction is *exothermic* or *endothermic* and whether it proceeds with small or large activation barrier. There are, however, situations where energies need to be replaced by Gibbs energies in order to take proper account contributions due to entropy.\*\* For example, a proper account of the equilibrium concentrations of reactants and products requires calculation of the equilibrium constant,  $K_{\text{eq}}$ , which according to the Boltzmann equation, is related to the Gibbs energy of reaction,  $\Delta G_{\text{rxn}}$ :

$$K_{\text{eq}} = \exp(-\Delta G_{\text{rxn}}/RT)$$

Here  $R$  is the gas constant and  $T$  is the temperature (in K). At room temperature (298K) and for  $\Delta G_{\text{rxn}}$  in au, this is given by:

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\* Referred to separated reactants, "negative activation energies" are possible. This is due to the formation of a complex, the total energy of which is lower than the sum of the total energies of the reactants. Also note that some reactions proceed with zero activation energy, meaning that there is no transition state.

\*\* Entropy contributions will be largest where the numbers of reactants and products differ. On the other hand, entropy might be expected to largely cancel where reactants and products are very similar.

$$K_{eq} = \exp(-1060 \Delta G_{rxn})$$

$\Delta G_{rxn}$  has two components, the enthalpy of reaction,  $\Delta H_{rxn}$ , and the entropy of reaction,  $\Delta S_{rxn}$ . These are defined as follows:

$$\Delta G_{rxn} = \Delta H_{rxn} - T\Delta S_{rxn}$$

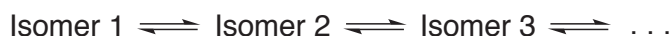
$$\Delta H_{rxn} \approx \Delta E_{rxn} = E_{product\ 1} + E_{product\ 2} + \dots - E_{reactant\ 1} - E_{reactant\ 2} - \dots$$

$$\Delta S_{rxn} = S_{product\ 1} + S_{product\ 2} + \dots - S_{reactant\ 1} - S_{reactant\ 2} - \dots$$

Although  $\Delta G_{rxn}$  depends on both enthalpy and entropy, it is often assumed that the entropy contribution will be small, and can be neglected. Further assuming that  $\Delta H_{rxn} \approx \Delta E_{rxn}$ , equilibrium constants can then be estimated according to the “Boltzmann” equation:

$$K_{eq} \approx \exp(-\Delta E_{rxn}/RT) \approx \exp(-1060 \Delta E_{rxn})$$

This equation may also be used to establish the equilibrium composition of a mixture of isomers:



$$\% \text{ Isomer } i = \frac{100 \exp(-1060 E_{\text{Isomer } i})}{\sum_k \exp(-1060 E_{\text{Isomer } k})}$$

Isomer energies,  $E_{\text{isomer}}$ , are given in atomic units relative to the energy of the lowest-energy isomer. An important special case involves an equilibrium between two isomers:



$$\frac{[\text{Isomer 1}]}{[\text{Isomer 2}]} = \exp[-1060 (E_{\text{isomer1}} - E_{\text{isomer2}})]$$

Reaction rate constants,  $k_{rxn}$ , are also related to Gibbs energies. As before, if entropy contributions can be neglected, the rate constant can be obtained directly from the activation energy,  $\Delta E^\ddagger$ , according to the Arrhenius equation:

$$k_{rxn} \approx (k_B T/h) [\exp(-\Delta E^\ddagger/RT)]$$

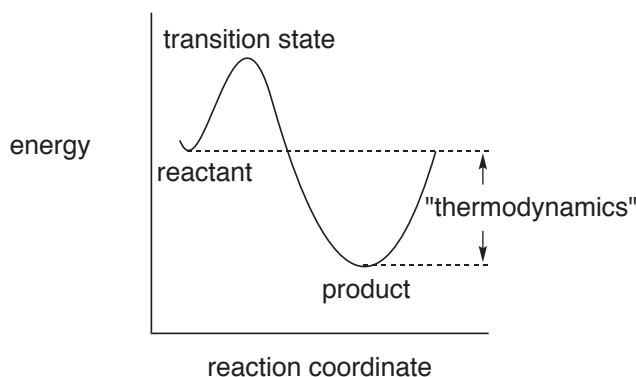
Here  $k_B$  and  $h$  are the Boltzmann and Planck constants, respectively. At room temperature and for  $\Delta E^\ddagger$  in au,  $k_{rxn}$  is given by:

$$k_{\text{rxn}} = 6.2 \times 10^{12} \exp(-1060 \Delta E^\ddagger)$$

Another way to describe reaction rates is by half-life,  $t_{1/2}$ , the amount of time it takes for the reactant concentration to drop to one half of its original value. When the reaction follows a first-order rate law,  $\text{rate} = -k_{\text{rxn}}[\text{reactant}]$ ,  $t_{1/2}$  is given by:

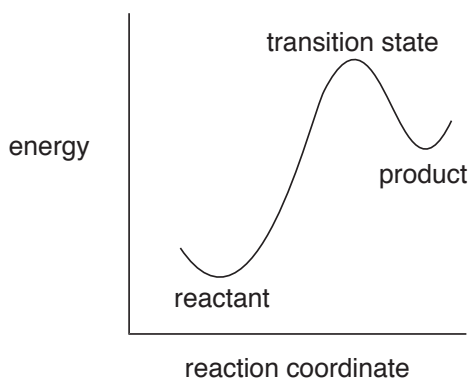
$$t_{1/2} = \ln 2 / k_{\text{rxn}} = 0.69 / k_{\text{rxn}}$$

It is useful to associate reaction energies and reaction rates with potential energy diagrams introduced earlier. The thermodynamics of reaction is given by the relative energies of the reactant and product on the potential surface.

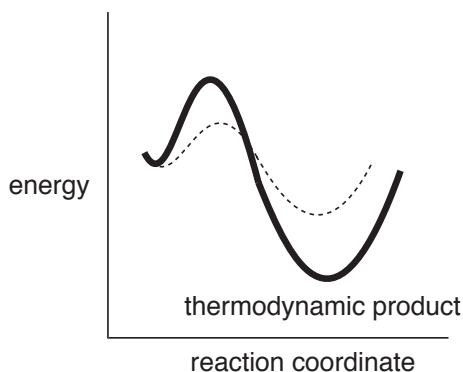


For situations such as bond rotation in ethane and ring inversion in cyclohexane, the reactant and product are the same and the reaction is said to be ***thermoneutral***. The most common case, as depicted in the above diagram, is where the energy of the products is lower than that of the reactants. The reaction is said to be *exothermic*, and the difference in stabilities of reactant and product is simply the difference in their energies. For example, the “reaction” of *gauche n*-butane to *anti n*-butane is *exothermic*, and the difference in stabilities of the two conformers is simply the difference in the energies ( $\sim 3.8$  kJ/mol).

Chemical reactions can also be *endothermic*, which give rise to a reaction profile.

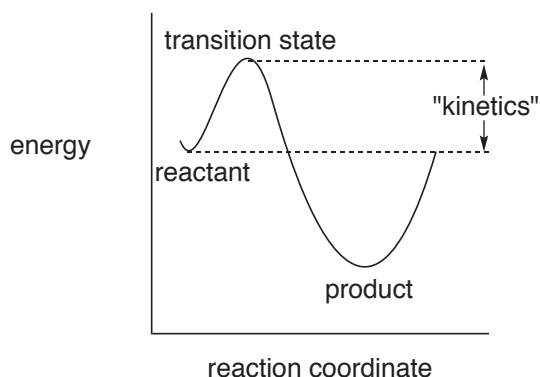


Where two or more different products may form in a reaction, thermodynamics tells us that if we wait long enough, the product formed in greatest abundance will be that with the lowest energy irrespective of pathway.



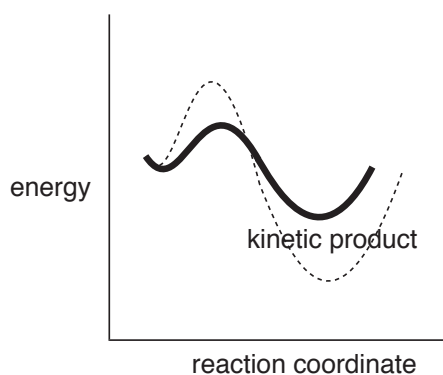
In this case, the product is referred to as the ***thermodynamic product*** and the reaction is said to be ***thermodynamically controlled***.

The energy of the transition state above the reactants (the activation energy) provides the connection with reaction rate (kinetics). Of course, such an interpretation is based on the notion that all reactions actually pass through a single transition state.



Aside from the obvious dependence on the concentration of reactants, absolute reaction rates also depend on the percentage that encounters between molecules will actually lead to reaction (the so-called pre-exponential or A factor). The hope is that pre-exponential factors for closely-related reactions will be very similar and therefore can be ignored. Thus, it is not at all evident that activation energies for disparate reactions will parallel measured reaction rates.

The product formed in greatest amount in a kinetically controlled reaction (the kinetic product) is that proceeding via the lowest energy transition state, irrespective of whether or not this is lowest energy product (the thermodynamic product).



Kinetic product ratios show dependence with activation energy differences which are identical to thermodynamic product ratios with difference in reactant and product energies.